

Taming galileons in curved spacetime

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Abstract

Localising the galileon symmetry along with Poincare symmetry we have found a version of galileon model coupled with curved spacetime which retains the internal galileon symmetry in covariant form. Also, the model has second order equations of motion.

The effective field theory approach has provided new insights and results in various contexts. Since these theories are defined by appropriate symmetries, new possibilities are always being suggested and probed. Over the last few years the viable symmetries of systems consisting of one or more scalar fields coupled to gravity have been explored due to their relevance in cosmological model building. This work was triggered by the introduction of a scalar field theory, called galileon [1]. It was obtained by taking the decoupling limit of the DGP model [2] leading to an effective scalar theory argued to describe the scalar sector of the original model [3, 4].

Galileon theories are interesting in several ways. They are characterised by two properties :

1. Despite being higher derivative theories, they are free of Ostrogradsky-type ghosts since they have no more than second order equations of motion.
2. They are invariant under a nonlinear symmetry transformation of the scalar field π ,

$$\pi \rightarrow \pi + c + b_\mu x^\mu \quad (1)$$

where c and b_μ are constants. A direct consequence of this invariance is that the coefficients of the leading galileon interactions are, perturbatively, not renormalised [5].

In four dimensions, there are five different parts $\mathcal{L}_i, i = 1, 2, \dots, 5$, which are complete in themselves. $\mathcal{L}_1 = \pi$ is trivial and $\mathcal{L}_2 = \partial_\mu \pi \partial^\mu \pi$ is the usual quintessence. The first nontrivial Galileon term is

$$\mathcal{L}_3 = \square \pi (\partial \pi)^2 \quad (2)$$

Despite its attractive features, the coupling of galileons to gravity is a tricky and challenging issue. A simple minimal coupling leads to a breakdown of the first property. It is possible to introduce a nonminimal coupling [6] which, however, does not retain the second property. In any case the symmetry transformation (1), as it stands, is meaningless in curved space. This has resulted in a general scheme for coupling scalar theories to gravity, incorporating only the first property [7]. Attempts to covariantize the condition (1) and define galileons in curved space have always led to restrictions [8, 9], so that the findings are not universal. In fact the general consensus is that it is impossible to couple galileons to gravity while retaining (1) or its curved space extension [10]. This has resulted in the introduction of models where galileon symmetry (1) is weakly broken [10].

In this paper we provide a new approach of coupling galileons to gravity where both properties are incorporated. This approach is based on the concept of localising a global symmetry, an example of which is (1). It was used earlier [11, 12] to localise the Poincare symmetry to provide an alternative formulation of gravity, called the Poincare gauge theory of gravity. Very recently in a set of papers [13, 14, 15, 16, 17], we have in collaboration with Mitra, developed a nonrelativistic version of it, the so called galilean gauge theory, by localising the galilean symmetry. Apart from yielding results related to nonrelativistic spatial diffeomorphism invariance, it gave a dynamical realisation of Newton-Cartan geometry, with or without torsion, which is the basis of nonrelativistic gravity. By adopting the same algorithm, we gauge (i.e, localise) both the Poincare symmetry and the symmetry (1) to consistently couple galileon scalars to gravity. Interestingly, the new fields introduced to gauge the galileon symmetry transform in a similar manner as the galileon generators that realise the galileon algebra [18].

The problem may be posed in the following way. We have a theory in the

flat space with the generic action,

$$\int d^4x \mathcal{L}(\pi, \partial_\mu \pi, \partial_\mu \partial_\nu \pi) \quad (3)$$

which is invariant under the combined Poincare and Galileon transformations,

$$\begin{aligned} \delta x_\mu &= \xi_\mu \\ \delta \pi &= -\xi^\lambda \partial_\lambda \pi + c + b_\mu x^\mu \\ \delta \partial_\mu \pi &= -\xi^\lambda \partial_\lambda \partial_\mu \pi + \theta_\mu{}^\lambda \partial_\lambda \pi + b_\mu \\ \delta \partial_\mu \partial_\nu \pi &= -\xi^\lambda \partial_\lambda \partial_\mu \partial_\nu \pi + \theta_\mu{}^\lambda \partial_\lambda \partial_\nu \pi + \theta_\nu{}^\lambda \partial_\lambda \partial_\mu \pi \end{aligned} \quad (4)$$

Here, $\xi^\lambda = \epsilon^\lambda + \theta^\lambda{}_\rho x^\rho$ is the infinitesimal Poincare transformation parameter. What are the modifications to be done in (3) and (4) when gravity is included? This is the principal aim of the paper.

The algorithm already stated requires a gauging of the original (global) symmetry. We wish to construct an action, starting from (3), that is invariant under the local version of (4). By localisation of the symmetry we mean that the transformation parameters $\epsilon^\mu, \theta^\mu{}_\nu, c, b_\mu$ become functions of spacetime. While this preserves the functional form for the field transformation,

$$\delta \pi = -\xi^\lambda \partial_\lambda \pi(x) + c(x) + b_\mu(x) x^\mu \quad (5)$$

there is an obvious nontrivial change in the transformation of the derivatives, e.g.,

$$\delta \partial_\mu \pi = -\xi^\lambda \partial_\lambda \partial_\mu \pi - (\partial_\mu \epsilon^\lambda + (\partial_\mu \theta^\lambda{}_\rho) x^\rho) \partial_\lambda \pi + \partial_\mu c + \theta_\mu{}^\lambda(x) \partial_\lambda \pi + b_\mu(x) + (\partial_\mu b_\sigma) x^\sigma \quad (6)$$

Coming back to the original flat space models, the Galileon transformations have already been defined in (1). The symmetry under (4) is due to the structure of the transformations of $\pi, \partial_\mu \pi$ and $\partial_\mu \partial_\nu \pi$. Together they ensure that the total change of \mathcal{L} is given by ¹,

$$\Delta \mathcal{L} = \delta \mathcal{L} + \xi^\lambda \partial_\lambda \mathcal{L} + \partial_\lambda \xi^\lambda \mathcal{L} = 2 \partial^\lambda \left[\left(\partial_\lambda \pi \partial_\mu \pi - \frac{1}{2} \eta_{\mu\lambda} (\partial_\alpha \pi \partial^\alpha \pi) \right) \right] b^\mu \quad (7)$$

where, $\delta \mathcal{L}$ is the form variation of \mathcal{L} ,

$$\delta \mathcal{L} = \mathcal{L}'(x) - \mathcal{L}(x) \quad (8)$$

¹For the sector involving only the Poincare symmetry, $\Delta \mathcal{L} = 0$.

We observe that the invariance is at the quasi level because $\Delta\mathcal{L}$ does not vanish but changes by a partial derivative. This quasi invariance is due to the galileon invariance and rests on

$$\partial_\mu b_\nu = 0 \quad (9)$$

In order to construct an action that is invariant under the local symmetry (5, 6) we have to introduce a local coordinate system, labelled by $a \equiv (0, A)$. They are trivially connected with the global coordinates $\mu \equiv (0, i)$,

$$\mathbf{e}_a = \delta_a^\mu \mathbf{e}_\mu \quad (10)$$

Later on we will see that how this connection becomes nontrivial.

Our task is then clear. We have to replace $\partial_\mu \pi$ and $\partial_\mu \partial_\nu \pi$ by their local counterparts such that they transform form invariantly as in (4). In the more familiar context of Poincare invariance only, the covariant derivatives $\nabla_a \pi$ and $\nabla_a \nabla_b \pi$ are defined in the following form [12, 19]

$$\nabla_a \pi = \Sigma_a^\mu D_\mu \pi; \quad D_\mu \pi = \partial_\mu \pi + \frac{1}{2} B_\mu^{ab} \sigma_{ab} \pi \quad (11)$$

$$\nabla_a \nabla_b \pi = \Sigma_a^\mu D_\mu (\Sigma_b^\nu D_\nu \pi) \quad (12)$$

where σ_{ab} is the Lorentz spin matrix whose form is dictated by the spin of the field π . Here Σ_a^μ and B_μ^{ab} are new gauge fields corresponding to translation and lorentz transformation². The galileon field has an additional shift symmetry. To compensate this, we require to introduce further gauge fields A_μ and D . The new covariant derivatives are now defined as,

$$\bar{\nabla}_a \pi = \Sigma_a^\mu \bar{D}_\mu \pi; \quad \bar{D}_\mu \pi = (D_\mu \pi + F_\mu; F_\mu = A_\mu + x_\mu D) \quad (13)$$

The transformations of the new fields are obtained by demanding that the covariant derivatives (13) transform as the ordinary one in (4),

$$\delta(\bar{\nabla}_a \pi) = -\xi^c \partial_c (\bar{\nabla}_a \pi) + \theta_a^b \bar{\nabla}_b \pi + b_a \quad (14)$$

This yields,

$$\begin{aligned} \delta \Sigma_a^\mu &= -\xi^\lambda \partial_\lambda \Sigma_a^\mu + \partial_\lambda \xi^\mu \Sigma_a^\lambda + \theta_a^b \Sigma_b^\mu \\ \delta B_\mu^{ab} &= -\xi^\lambda \partial_\lambda B_\mu^{ab} - \partial_\mu \theta^{ab} - \partial_\mu \xi^\lambda B_\mu^{ab} + \theta^a_c B_\mu^{cb} + \theta^b_c B_\mu^{ac} \\ \delta A_\mu &= -\xi^\lambda \partial_\lambda A_\mu - \partial_\mu \xi^\lambda A_\lambda - \partial_\mu c \\ \delta(x_\mu D) &= -\xi^\lambda \partial_\lambda (x_\mu D) - \partial_\mu \xi^\lambda (x_\lambda D) - x^\nu \partial_\mu b_\nu \end{aligned} \quad (15)$$

²The gauge fields corresponding to the Lorentz rotation do not appear because π is a Lorentz scalar.

We observe that A_μ and $x_\mu D$ transform as four vectors under local Poincare transformations. Galileon transformations, on the other hand, act like independent translations. The Galileon symmetries manifested through these relations are compatible with analogous results in [18] where the algebra of Galileon generators has been defined. Also note that

$$\begin{aligned}\delta \bar{D}_\mu \pi &= -\xi^\lambda \partial_\lambda (\bar{D}_\mu \pi) - \partial_\mu \xi^\lambda \bar{D}_\lambda \pi + b_\mu \\ \delta F_\mu &= -\xi^\lambda \partial_\lambda F_\mu - \partial_\mu \xi^\lambda F_\lambda - \partial_\mu c - x^\nu \partial_\mu b_\nu\end{aligned}\quad (16)$$

Thus both F_μ and $\bar{D}_\mu \pi$ transform as four vectors under local Poincare transformations.

The galileon model (2) also contains second derivative of π . The transformations of the second derivatives, however, do not have any galileon contribution (see the last eq. of (4)). Consequently, the second derivative $\partial_\mu \partial_\nu \pi$ should be replaced by $\nabla_a \bar{\nabla}_b \pi$ ³. Explicit calculation shows that under local Poincare plus galileon transformations $\nabla_a \bar{\nabla}_b \pi$ transform in the same way as $\partial_\mu \partial_\nu \pi$ i.e.

$$\delta(\nabla_a \bar{\nabla}_b \pi) = -\xi^d \partial_d (\nabla_a \bar{\nabla}_b \pi) + \theta_a^d (\nabla_d \bar{\nabla}_b \pi) + \theta_b^d (\nabla_a \bar{\nabla}_d \pi) \quad (17)$$

provided,

$$\nabla_a b_c = 0 \quad (18)$$

This is a natural generalization of the condition (9) in flat space.

An important consequence of (7), to be exploited later, is now discussed. Defining F_a from F_μ as,

$$F_a = \Sigma_a^\mu F_\mu \quad (19)$$

and,

$$\nabla_a F_b = \Sigma_a^\mu [\partial_\mu (\Sigma_b^\nu F_\nu) + B_\mu^{bc} \Sigma_c^\nu F_\nu] \quad (20)$$

we find,

$$\nabla_a F_b - \nabla_b F_a = \Sigma_a^\mu \Sigma_b^\nu (\partial_\mu F_\nu - \partial_\nu F_\mu) \quad (21)$$

for zero torsion,

$$\nabla_a \Sigma_c^\mu - \nabla_c \Sigma_a^\mu = 0 \quad (22)$$

³A general discussion of localising the Poincare sector of a higher derivative theory is given in [19].

The variation under galileon transformations now yields,

$$\begin{aligned}\delta(\nabla_a F_b - \nabla_b F_a) &= \Sigma_a^\mu \Sigma_b^\nu (\partial_\mu b_\nu - \partial_\nu b_\mu) \\ &= (\nabla_a b_b - \nabla_b b_a) = 0\end{aligned}\tag{23}$$

on account of (18). This implies that the choice,

$$(\nabla_a F_b - \nabla_b F_a) = 0\tag{24}$$

may be consistently implemented.

When the transformations are localised the theory invariant under local transformations is obtained by replacing the first order and second order derivatives by the corresponding covariant derivatives. Coming back to the Galileon action, we write its localised form as

$$S = \int d^4x \frac{1}{\Sigma} \bar{\mathcal{L}}_3 = \int d^4x \frac{1}{\Sigma} [(\nabla_a \bar{\nabla}^a \pi)((\bar{\nabla}_b \pi)(\bar{\nabla}^b \pi)]\tag{25}$$

where $\Sigma = \det \Sigma_a^\mu$ and the factor Σ^{-1} corrects the change due to the fact that, contrary to the global case, $\partial_\mu \xi^\mu \neq 0$ (see (7)).

Now it is known that if an action is invariant under the usual (global) transformations, then under local transformations the invariance is recovered by suitably replacing the ordinary derivatives by the covariant derivatives. The same is however not true if the action is only quasi invariant, as happens here. Nevertheless, we proceed with the same algorithm and finally make an explicit check of the invariance.

The theory (25) is a theory in the flat space but the transformations of the fields suggest a lucrative geometric interpretation. The action (25) may be viewed as the first order (vielbein) formulation of the galileon model in curved space time. In this interpretation Σ_a^μ is the tetrad and B_μ^{ab} are the spin connections [12].

To identify (25) as the required Galileon model we have to show that the properties 1 and 2 hold. It is easy to show that the higher (third) order derivatives cancel in the corresponding Euler Lagrange equations. Hence (25) is ghost free. To prove that the shift symmetry holds, we first calculate the variation of $\bar{\mathcal{L}}_3$ from (25),

$$\delta \bar{\mathcal{L}}_3 = 2 \nabla_a \left[\bar{\nabla}^a \pi \bar{\nabla}_c \pi b^c - \frac{1}{2} \delta_c^a (\bar{\nabla} \pi)^2 b^c \right] - 2 (\bar{\nabla}^a \pi) b^c (\nabla_a F_c - \nabla_c F_a)\tag{26}$$

Using the condition (24), the quasi invariance of the galileon action (25) is proved.

Thus, our goal has been achieved. By following a systematic algorithm we have gauged the original (global) Poincare and galileon symmetries of the model (2). The new model (25) does not lead to any higher order equations of motion and is also (quasi) invariant under (5), which is interpreted as the appropriate covariant version of the galileon symmetry (4). The passage to the flat limit is smooth and is simply obtained by setting the new fields ($\Sigma_a{}^\mu$, $B_\mu{}^{ab}$, A_μ and D) to zero. Although explicit results were given for \mathcal{L}_3 (2), the method is generic and easily applicable to other galileon invariant models.

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